Mortality Pattern at Adult and Older Ages

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Life-table for Life Expectancy

- Life tables summarize mortality information
- Constant-hazard assumption in the open-ended age group
- Huge portion of the population surviving to this age

Handling censoring adequately: gamma-Gompertz-Makeham model
Gompertz Baseline

▶ Force of mortality for an individual:

Gompertz

\[ \mu_G(x) = ae^{bx} \]

Gompertz-Makeham

\[ \mu_{GM}(x) = ae^{bx} + c \]

▶ Frailty:

Gamma-Gompertz

\[ \mu_{\Gamma G}(x \mid z) = zae^{bx} \]

Gamma-Gompertz-Makeham

\[ \mu_{\Gamma GM}(x \mid z) = zae^{bx} + c \]

\[ Z \sim \Gamma(1/\gamma, 1/\gamma), \ E(Z) = 1, \ CV^2(Z) = \gamma \]

▶ Force of mortality for the population:

\[ \mu_{\Gamma G}(x) = \frac{ae^{bx}}{1 + \frac{a\gamma}{b}(e^{bx} - 1)} \]

\[ \mu_{\Gamma GM}(x) = \frac{ae^{bx}}{1 + \frac{a\gamma}{b}(e^{bx} - 1)} + c \]

[Gompertz, 1825, Makeham, 1860, Vaupel et al., 1979]
Hazards with Gompertz Baseline

Parameter values: $a=0.00001$, $b=0.14$, $c=0.001$, $\gamma=0.2$
Why the gamma-Gompertz-Makeham model

- Captures both excess mortality at young-adult ages and the deceleration of death rates at older ages
- The model is able to capture both an infinitely increasing risk of death and an S-shaped pattern
- Qualitatively similar to the Kannisto model applied by the largest high-quality mortality databases

Kannisto model:
\[ \mu_K(x) = \frac{a e^{b(x-x_0)}}{1+a e^{b(x-x_0)}} \]
Advantages over Kannisto

- unobserved heterogeneity - frailty (parameter gamma)
- flexibility of capturing Gompertz whereas Kannisto always predicts mortality plateau
- the magnitude of the plateau is not restricted to 1
- Makeham term - less sensitive to the starting age of analysis.
- can be fitted over wider age range
- expansion of abridged (grouped) life-table values to (non-)integer ages.
- tails of the distribution, good fit
- parameters are easily interpreted, clear demographic meaning, input for further calculations
Censoring assumption

Raw data
Life table
ΓGM model
censoring assumptions

A
B

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## Reconstructed Life Expectancy

<table>
<thead>
<tr>
<th>Region</th>
<th>Year</th>
<th>Gender</th>
<th>Life Expectancy at Birth</th>
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<tr>
<td>Ache</td>
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<td>n/a</td>
<td>n/a</td>
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<tr>
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<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Hiwi</td>
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<td>n/a</td>
<td>n/a</td>
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<td>n/a</td>
<td>n/a</td>
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<tr>
<td>Bangladesh</td>
<td>1974 F</td>
<td>F</td>
<td>40</td>
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<tr>
<td>India</td>
<td>1986 M</td>
<td>M</td>
<td>60</td>
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<tr>
<td>Colombia</td>
<td>2005 F</td>
<td>F</td>
<td>70</td>
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<td>Rep. of Korea</td>
<td>1995 M</td>
<td>F</td>
<td>75</td>
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<td>Sri Lanka</td>
<td>2000−2002 M</td>
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<td>80</td>
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<td>Malta</td>
<td>2007 M</td>
<td>F</td>
<td>85</td>
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<tr>
<td>Mongolia</td>
<td>1996−2000 F</td>
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<td>90</td>
</tr>
</tbody>
</table>

Life table with constant hazard
ΓGM
Siler

Hunter–gatherer

Contemporary
Life Expectancy Reconstruction

- **Life-table Databases**
  - Human Mortality Database (HMD)
  - Human Life-table Database (HLD)
  - Global Health Observatory Data Repository (WHO)
  - World Population Prospects (WPP)
  - Database published by Eurostat

- HMD, WHO, WPP use cubic splines smoothing + Kannisto model

- Method protocols do not give clear justification
Lifespan Inequality Measures

![Graph showing life expectancy at birth vs. life expectancy lost due to death for different measures: HLD, WPP, WHO, HMD.](chart.png)
References

