4th HMD Symposium

**Life Expectancy and Lifespan Equality: A Dynamic Long run Relationship**

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23\textsuperscript{rd} May 2017
Introduction

- **Background:**
  - Life expectancy at birth ($e_0$) is one of the most widely used measures to summarize population health.
  - Most countries have improved in this indicator. Record $e_0$ has steadily increased by 2.5 years every decade.
  - However, it conceals variation in lifespans or **lifespan equality**.
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- What is lifespan equality?
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What is lifespan equality?
- Dimension that expresses a fundamental difference in survivorship among individuals.
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  ▶ Most countries have improved in this indicator. Record $e_0$ has steadily increased by 2.5 years every decade.
  ▶ However, it conceals variation in lifespans or **lifespan equality**.

► **What is lifespan equality?**
  ▶ Dimension that expresses a fundamental difference in survivorship among individuals.
  ▶ It addresses the growing interest in health inequalities and its linkage with social behavior.
Why studying **lifespan equality** is important?

**Danish Females**

- **$e_0$**
  - 1945: 67.2
  - 2010: 81.33
- **IQR**
  - 1945: 19.99
  - 2010: 14.66
- **$\Omega_0$**
  - 1945: 89.99
  - 2010: 97.33

*Source: HMD 4th HMD Symposium Aburto et al. 2017*
Strong association between life expectancy and lifespan equality

Life expectancy ($e_0$) vs lifespan equality ($\eta$)

Pearson correlation coefficient > .95

Period
- 1900–1921
- 1921–1959
- 1960 onwards

Life Expectancy and Lifespan Equality

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Life Expectancy and Lifespan Equality
Non-stationary series

Life expectancy

Lifespan equality ($\eta$)
If non-stationarity $\rightarrow$ risk of misleading results

Life expectancy ($e_0$) vs lifespan equality ($\eta$)

R squared

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Sex
Females
Males
Stochastic properties suggest analyzing both in first differences

Changes in life expectancy and lifespan equality

Period
- 1900−1921
- 1921−1959
- 1960 onwards

R square = 0.704

(Δe₀)
(Δη)

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General idea of the model

Life expectancy ($e_0$) vs lifespan equality ($\eta$)
General idea of the model

Life expectancy ($e_0$) vs lifespan equality ($\eta$)
General idea of the model

Life expectancy \( (e_0) \) vs lifespan equality \( (\eta) \)

Life expectancy

Lifespan equality

Long term equilibrium
General idea of the model

Life expectancy ($e_0$) vs lifespan equality ($\eta$)

Life expectancy (Japan, Russia)

"Mortality forces" $-\alpha$

"Mortality forces" $\alpha$

Life Expectancy and Lifespan Equality
General idea of the model

Life expectancy ($e_0$) vs lifespan equality ($\eta$)

"Mortality forces" $\alpha$

Japan (♀2000)

Russia (♂1994)

"Mortality forces" $-\alpha$
General idea of the model

Life expectancy ($e_0$) vs lifespan equality ($\eta$)

Life expectancy

Lifespan equality

"Mortality forces"

$-\alpha$

$x_\infty \mid t$

"Mortality forces"

$\alpha$

$\alpha' \sum_{i=1}^{t} \varepsilon_i$

$\beta'x_{1994}$

$\beta'x_{2000}$
Cointegration analysis

Two-dimensional VAR model in its equilibrium correction (VECM) form:

\[
\Delta Z_t = \sum_{i=1}^{k-1} \Gamma \Delta Z_{t-i} + \alpha \beta' Z_{t-1} + \mu + \Psi D_t + \epsilon_t
\]

where:

- \( \Delta \) first difference operator
- \( Z_t \) vector of stochastic variables, \( e_0 \) and \( \eta \)
- \( D_t \) vector of deterministic variables (e.g. linear trends)

Data comes from HMD, over 8 500 lifetables for 44 countries
Lifespan equality measures

Three measures were used:

\[ \eta = - \log \left( \frac{- \int_0^\omega \ell(x) \ln \ell(x) dx}{\int_0^\omega \ell(x) dx} \right) = - \log \left( \frac{e^\dagger}{e_0^o} \right), \]  \hspace{1cm} (1)

\[ \bar{\ell} = - \log \left( 1 - \frac{- \int_0^\omega \ell^2(x) dx}{\int_0^\omega \ell(x) dx} \right) = - \log (G), \]  \hspace{1cm} (2)

\[ cv = - \log \left( \frac{\sqrt{\int_0^\omega (x - e_0^o)^2 f(x) dx}}{\int_0^\omega \ell(x) dx} \right) = - \log \left( \frac{\sigma}{e_0^o} \right), \]  \hspace{1cm} (3)

\( \eta \) based on Keyfitz' entropy used in Colchero et al 2016.

\( \bar{\ell} \) a variant of the Gini coefficient.

\( cv \) a variant of the coefficient of variation.
Long run relationship [Johansen’s trace test]
Speed of adjustment towards long term equilibrium

Life expectancy

Lifespan equality

Sex

Females

Males

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Include the age dimension Reducing deaths at any age increases $e_0$; for $\eta$, it depends whether deaths occur before or after $a^i$
Threshold age $a^n$
Decomposition method

**Model of continuous change**: analysis based on the assumption that covariates change continuously along an actual or hypothetical dimension. [Horiuchi et al. 2008 Demography; Caswell 2010 Journal of Ecology]

The effect of the $i$-th age group death rate on the change in $e_0$ and $\eta$ from period $t$ to $t + 1$ can be calculated as

$$c_i = \int_{m_i(t)}^{m_i(t+1)} \frac{\partial e_0(t)}{\partial m_i(t)} dm_i(t)$$  \hspace{1cm} (4)

Then we calculated contributions below and above the threshold age to changes in life expectancy and lifespan equality.
Age-specific contributions

Changes below the threshold age

Changes above the threshold age
Summary and conclusions

- Strong association between changes in $e_0$ and $\eta$.
- We found evidence of a long term equilibrium.
- Even if in the short term they diverge from each other, there is a correction mechanism that bring them together again.
- To some extent mortality improvements below threshold age are driving the relationship.
Thanks for your attention.

Comments and/or questions?
Normalized \((\eta = 1)\) long run coefficient for \(e_0\)
Can we talk about *causality*?

- Granger causality $\rightarrow$ Because $e_0$ and $\eta$ cointegrate at least Granger causality exists in one direction.[Caution!]
  - Just a potential causality, does not take into account latent variables.
  - Temporal precedence: a cause precedes its effects in time
- Instantaneous causality: test non-zero correlation between error processes of the cause and effect variables.
  In 90% of the cases we reject the $H_0 = \text{no instantaneous causality}$
**long run relationship**

![Graph showing long run relationship](image)

**Measure**

- \((1-G)\)
- \(\log(\text{inv}(H))\)
- \(\log(\text{inv}(CV))\)

**Distance from 95% critical value**

- Latvia
- Israel
- West Germany
- Norway
- Ukraine
- Taiwan
- Canada
- Belarus
- Denmark
- Finland
- Switzerland
- Lithuania
- Denmark
- Russia
- Poland
- Sweden
- New Zealand
- Australia
- UK
- Ireland
- Czech Republic
- Slovakia
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