

# Mortality Pattern at Adult and Older Ages

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5<sup>th</sup> HMD Symposium

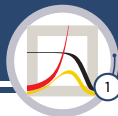
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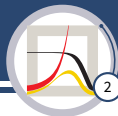


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RESEARCH



- ▶ Life tables summarize mortality information
- ▶ Constant-hazard assumption in the open-ended age group
- ▶ Huge portion of the population surviving to this age

Handling censoring adequately: gamma-Gompertz-Makeham model



- ▶ Force of mortality for an individual:

Gompertz

$$\mu_G(x) = ae^{bx}$$

Gompertz-Makeham

$$\mu_{GM}(x) = ae^{bx} + c$$

- ▶ Frailty:

Gamma-Gompertz

$$\mu_{\Gamma G}(x | z) = zae^{bx},$$

Gamma-Gompertz-Makeham

$$\mu_{\Gamma GM}(x | z) = zae^{bx} + c,$$

$$Z \sim \Gamma(1/\gamma, 1/\gamma), E(Z) = 1, CV^2(Z) = \gamma$$

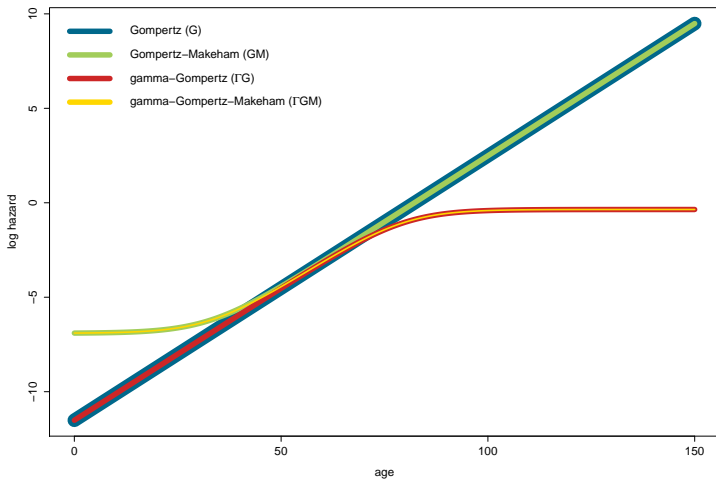
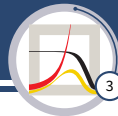
- ▶ Force of mortality for the population:

$$\mu_{\Gamma G}(x) = \frac{ae^{bx}}{1 + \frac{a\gamma}{b}(e^{bx} - 1)}$$

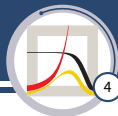
$$\mu_{\Gamma GM}(x) = \frac{ae^{bx}}{1 + \frac{a\gamma}{b}(e^{bx} - 1)} + c.$$

[Gompertz, 1825, Makeham, 1860, Vaupel et al., 1979]

# Hazards with Gompertz Baseline



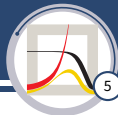
Parameter values:  $a=0.00001$ ,  $b=0.14$ ,  $c=0.001$ ,  $\gamma=0.2$



- ▶ Captures both excess mortality at young-adult ages and the deceleration of death rates at older ages
- ▶ the model is able to capture both an infinitely increasing risk of death and an S-shaped pattern
- ▶ qualitatively similar to the Kannisto model applied by the largest high-quality mortality databases

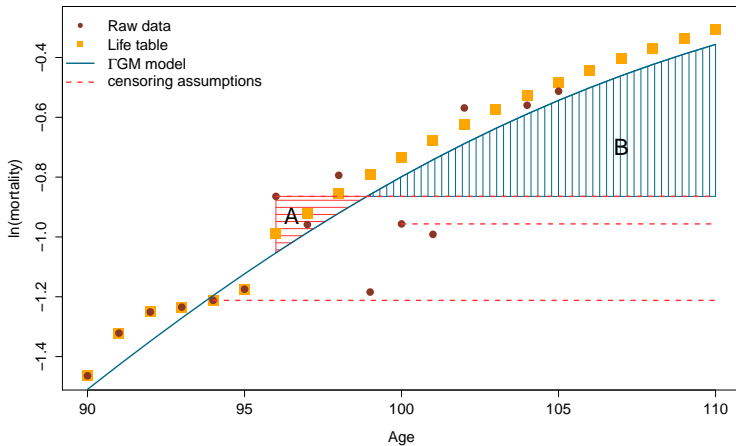
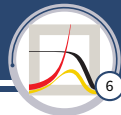
Kannisto model:

$$\mu_K(x) = \frac{ae^{b(x-x_0)}}{1+ae^{b(x-x_0)}}$$

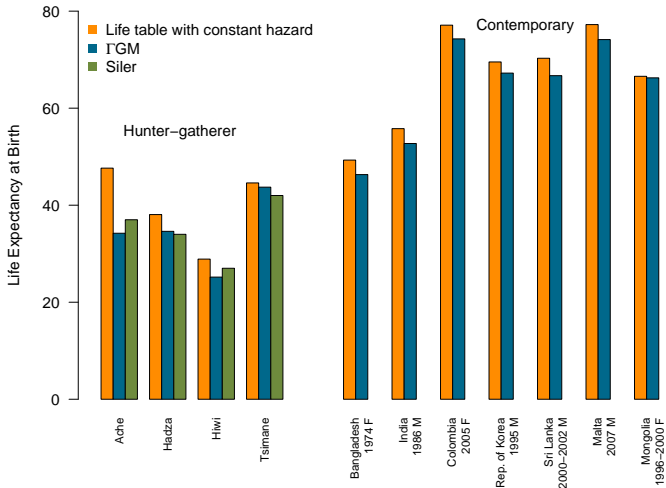
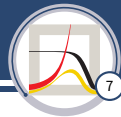


- ▶ unobserved heterogeneity - frailty (parameter gamma)
- ▶ flexibility of capturing Gompertz whereas Kannisto always predicts mortality plateau
- ▶ the magnitude of the plateau is not restricted to 1
- ▶ Makeham term - less sensitive to the starting age of analysis.
- ▶ can be fitted over wider age range
- ▶ expansion of abridged (grouped) life-table values to (non-)integer ages.
- ▶ tails of the distribution, good fit
- ▶ parameters are easily interpreted, clear demographic meaning, input for further calculations

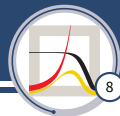
# Censoring assumption



# Reconstructed Life Expectancy

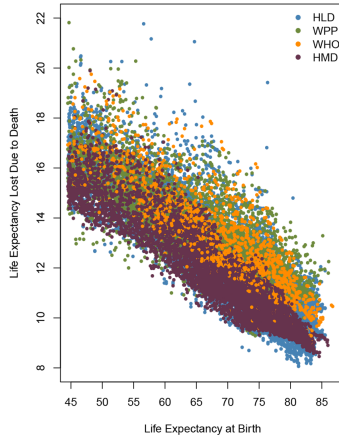
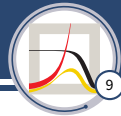


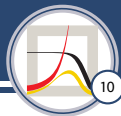




- ▶ Life-table Databases
  - ▶ Human Mortality Database (HMD)
  - ▶ Human Life-table Database (HLD)
  - ▶ Global Health Observatory Data Repository (WHO)
  - ▶ World Population Prospects (WPP)
  - ▶ Database published by Eurostat
- ▶ HMD, WHO, WPP use cubic splines smoothing + Kannisto model
- ▶ method protocols do not give clear justification

# Lifespan Inequality Measures





## References

- B. Gompertz. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. *Philosophical Transactions of the Royal Society of London*, 115:513–583, 1825.
- W.M. Makeham. On the law of mortality and the construction of annuity tables. *Journal of the Institute of Actuaries*, 8: 301–310, 1860.
- J.W. Vaupel, K.G. Manton, and E. Stallard. The impact of heterogeneity in individual frailty on the dynamics of mortality. *Demography*, 16:439–454, 1979.