Using Benford's law to assess life table ensembles

HMD and WHO Model Life Tables

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Overview

I will describe some work on quality assessment of life table ensembles.

This is work in progress

I will show how Benford's law can be used to perform quality control

This work is intended to be a tool in a toolbox, not the last word

Benford's law

Consider any collection of N numbers

Now consider only the first *significant* digit
[viz., omit leading zeros]

The distribution of N digits, $d_i \in \{1, 2, \ldots, 9\}$;
$i = 1..N$, is random (depending...)

but *not*, typically, uniform in natural or man-made data sets

Rather, d_i follows the Newcomb-Benford distribution (approximately)

Newcomb-Benford distribution

For base 10, the probability mass function of the Newcomb-Benford distribution is:

$$p(d) = \log_{10}\left(1 + \frac{1}{d}\right)$$

in which $p(d)$ is the proportion of values with first significant digit $d \in \{1, 2, \ldots, 9\}$

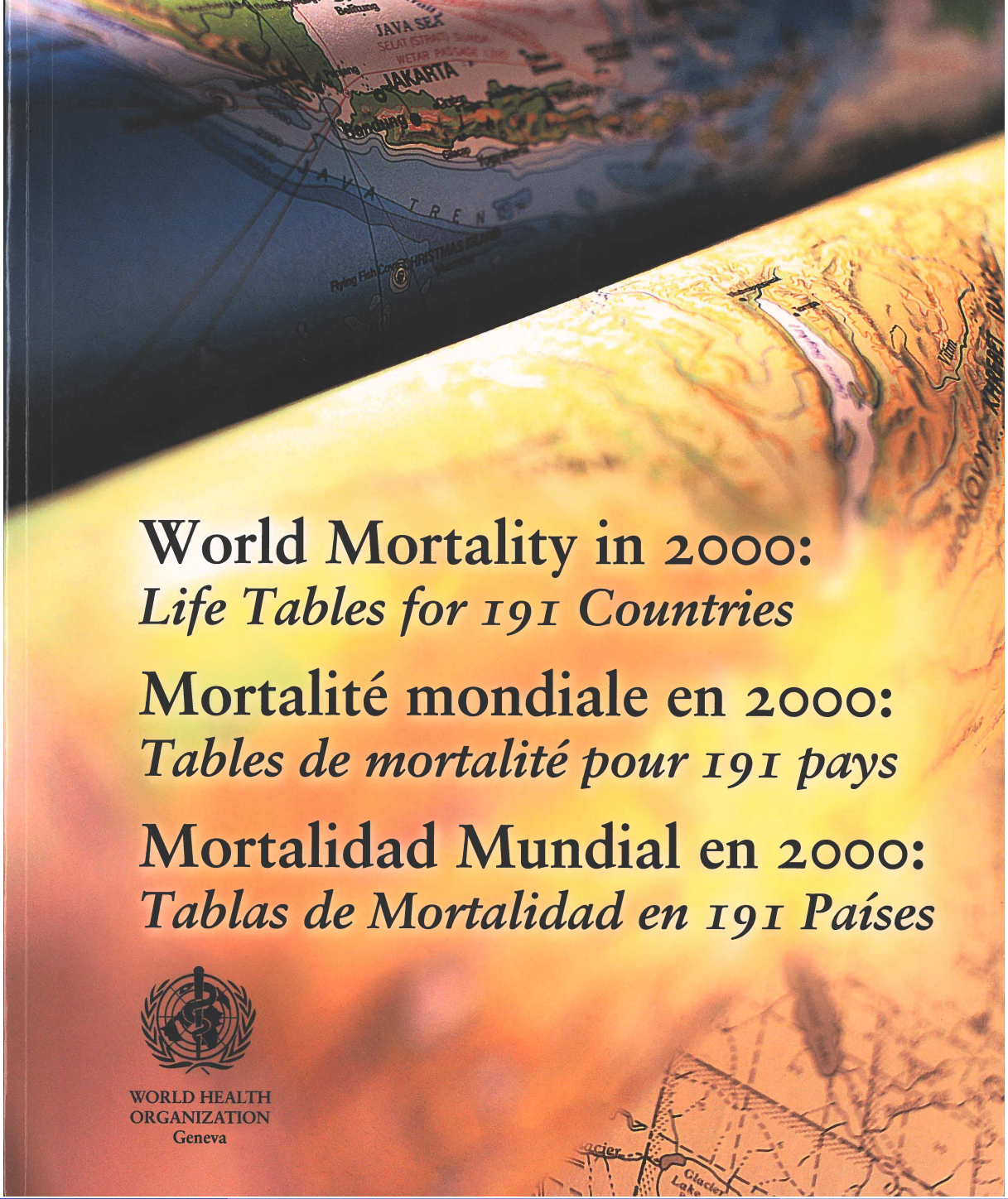
Agreement can be assessed via χ^2 test

(for small samples, see also Jann 2008; DOI: 10.1177/1536867X0800800201)

Databases

Mortality Databases considered here:

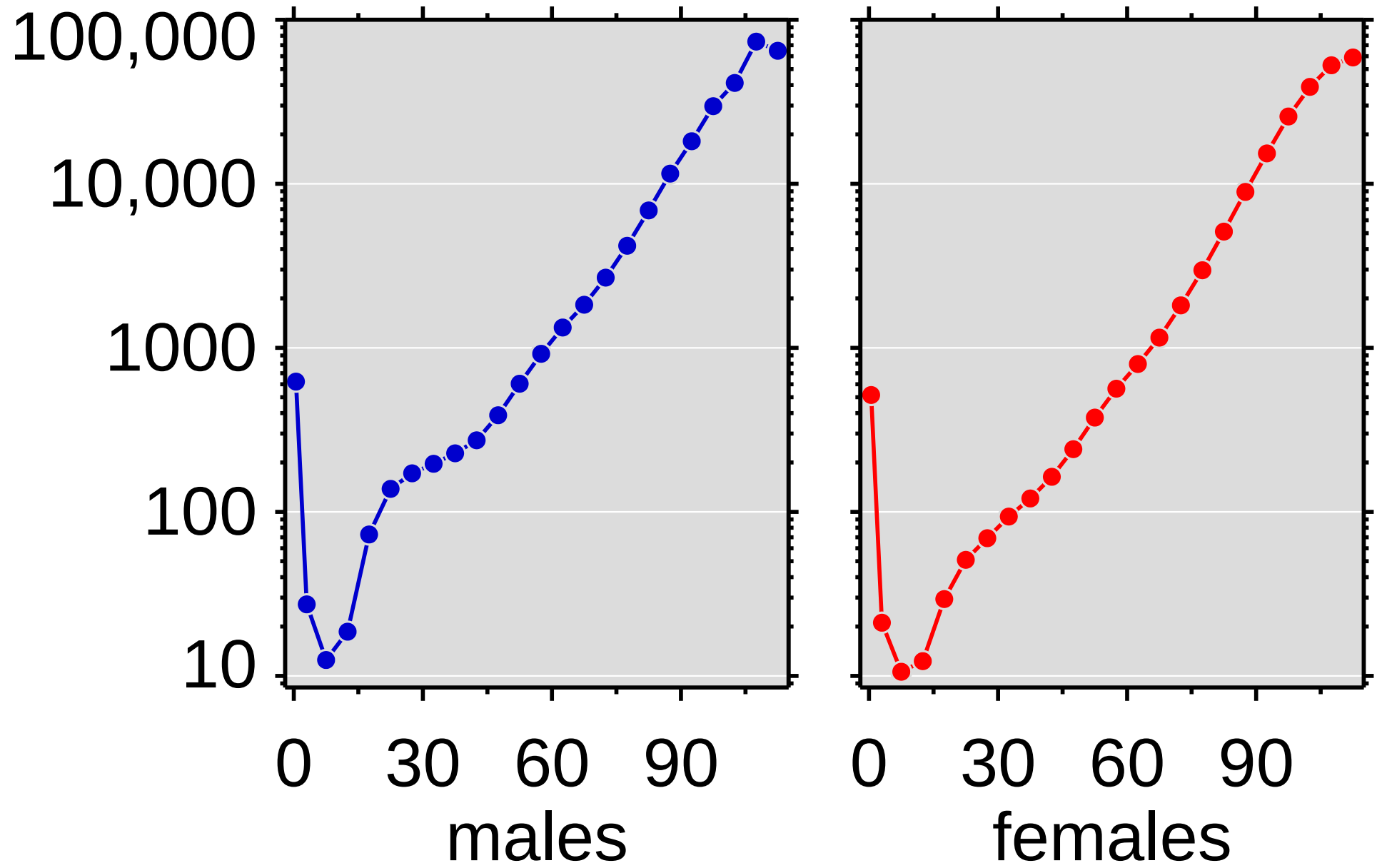
1) HMD
2) WHO year 2000 life tables (see next slide)


JAVA SEA
JAKARTA
Bandung
Semarang
JAVA TRENCH
Flying Fish Cove
CHRISTMAS ISLAND
INDIAN OCEAN
World Mortality in 2000:
Life Tables for 191 Countries
Mortalité mondiale en 2000:
Tables de mortalité pour 191 pays
Mortalidad Mundial en 2000:
Tablas de Mortalidad en 191 Países
WORLD HEALTH
ORGANIZATION
Geneva
Glacier
Glacier
Lake

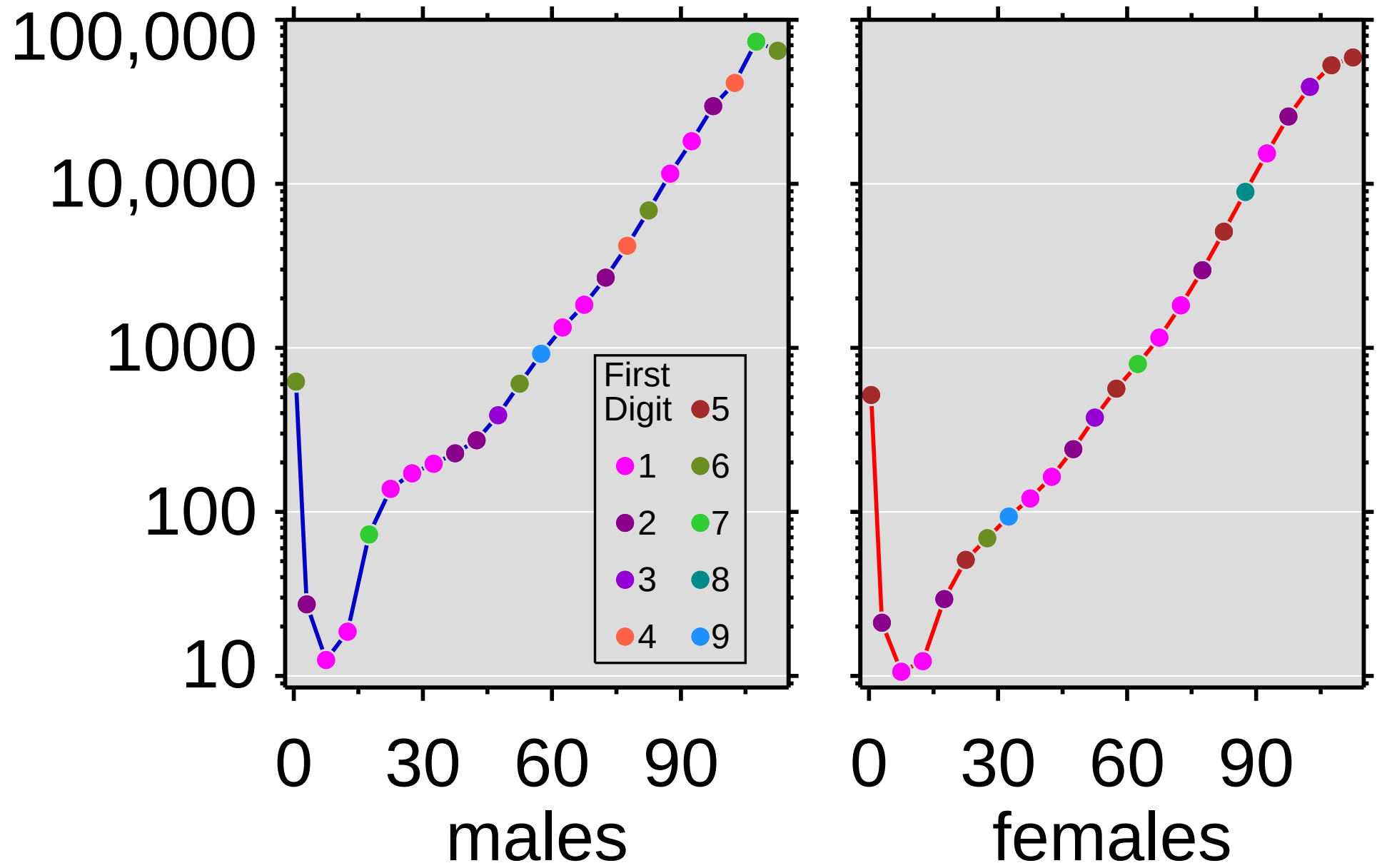
Approach

- The l.t. quantity I decided to investigate is ${}_nM_x$
- ℓ_x possibly less suitable?
- large sample: 222,288 digits (HMD), and 8,404 (WHO) [191 countries $\times$ 2 sexes $\times$ 22 age groups $\times$ 1 year]
- χ^2 test for deviation from N-B distribution
- digit histogram with N-B p.m.f. superposed ("Benfordgram")

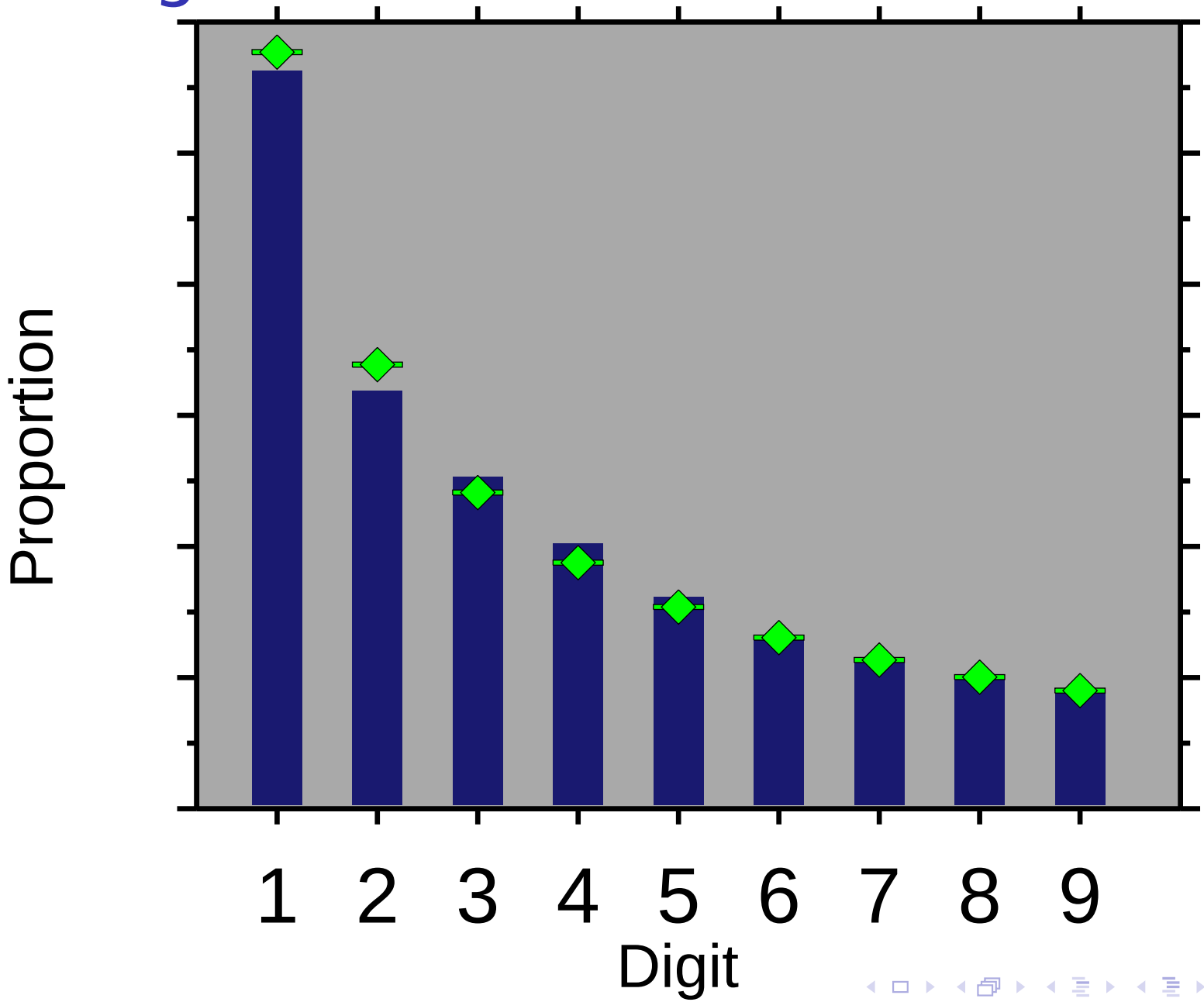
nMx (/100,000)



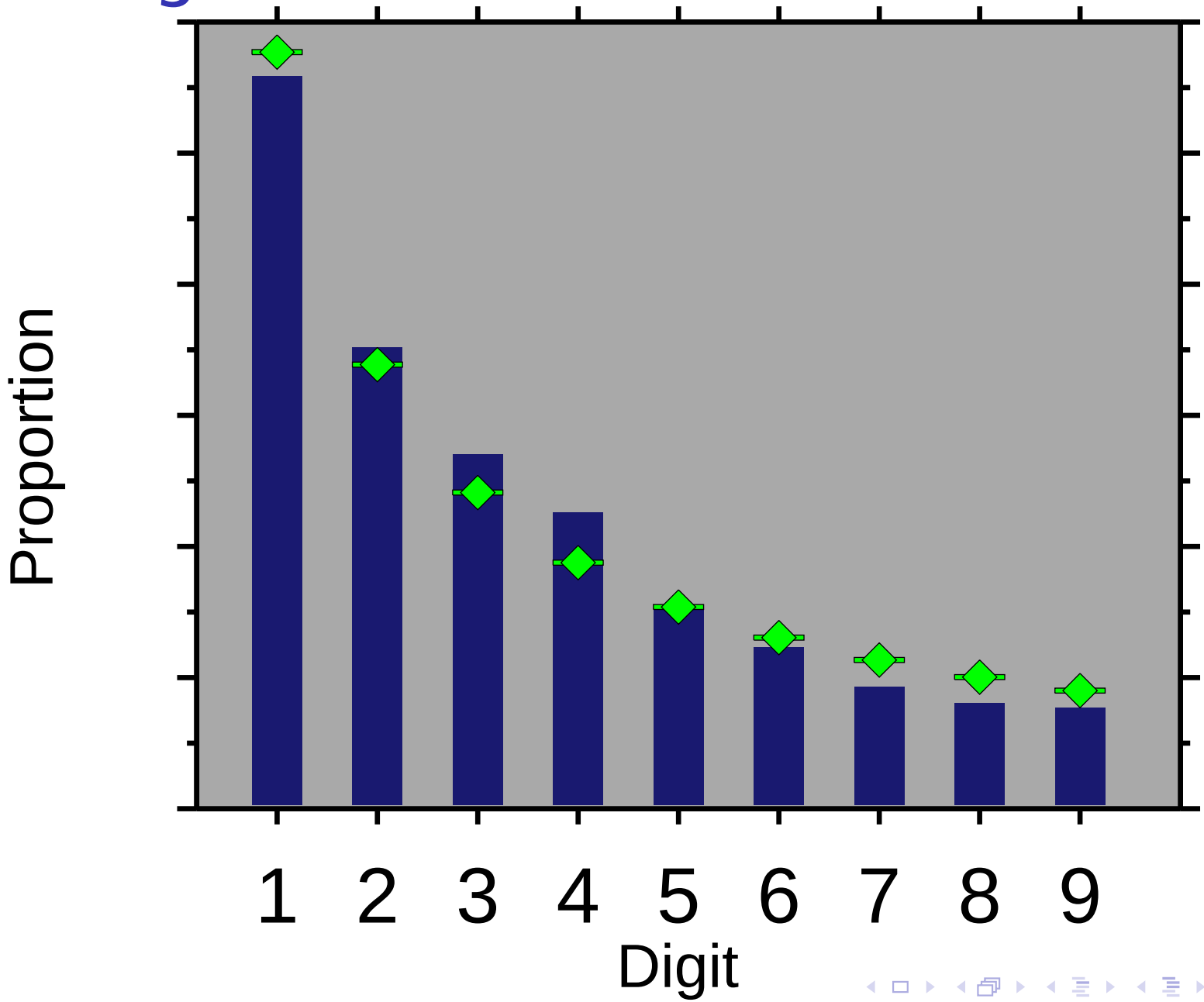
nMx (/100,000)


100,000
10,000
1000
100
10
First Digit
1
2
3
4
5
6
7
8
9
0
30
60
90
males
0
30
60
90
females

Benfordgram: HMD



Benfordgram: WHO



Highlights

- The Benfordgrams are probably the most important part
- Neither HMD or WHO are *statistically* N-B
- HMD is qualitatively pretty close to N-B: too few 1,2; too many 3,4,5; 6–9 spot-on.
- WHO: only 5 is spot on
- WHO has an additional "feature" that 8 and 9 very close (8: 4.1% ; 9: 3.9%) — which is a big deviation from N-B (i.e., levels off instead of decline)
- By comparison, HMD [$\doteq$N-B]: 8: 5.1% ; 9: 4.6%

In conclusion 1

This is work in progress, and I thank you for any feedback you may have

I believe Benford's law can be exploited as a quality check for life table ensembles

Neither the HMD or the WHO ${}_nM_x$ is an exact fit to the Newcomb-Benford distribution. WHO is worse.

A priori, we expect WHO data to be of poorer quality and more self-similar [191 countries], and the Benford's law analysis aligns with expectations

This suggests that Benford's law can, indeed, be used as quality control for life table ensembles.

Thank you

Thank you.

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