



Forecasting Deaths from a Single Cause with Competing Risks

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4th Human Mortality Database Symposium
Berlin, 22nd – 23rd May, 2017

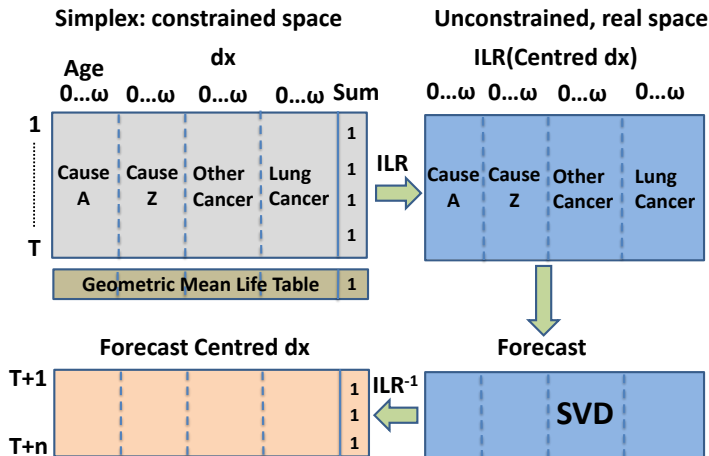
Outline

- 1 The Problem
- 2 The Data
- 3 Compositional Arithmetic
- 4 CoDa Lee–Carter and the SVD
- 5 Kullback–Leibler Divergence
- 6 Model Estimates
- 7 Forecast
- 8 Cause of Death Correlations

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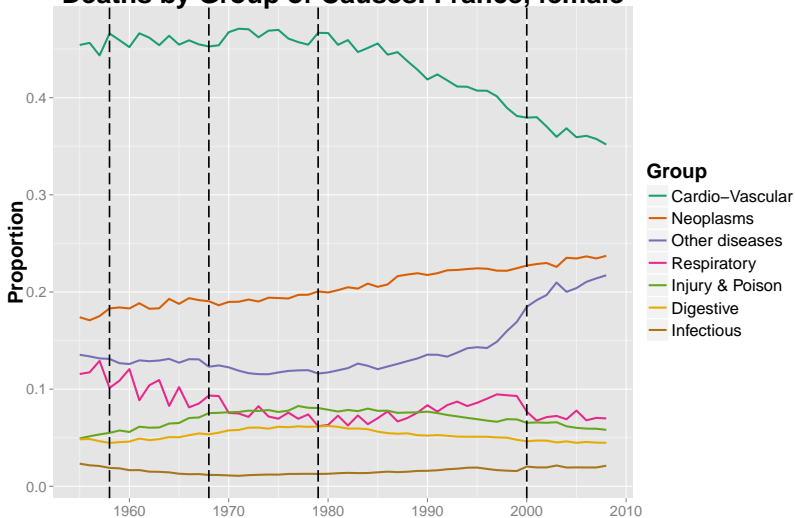
One Cause of Death and Competing Risks

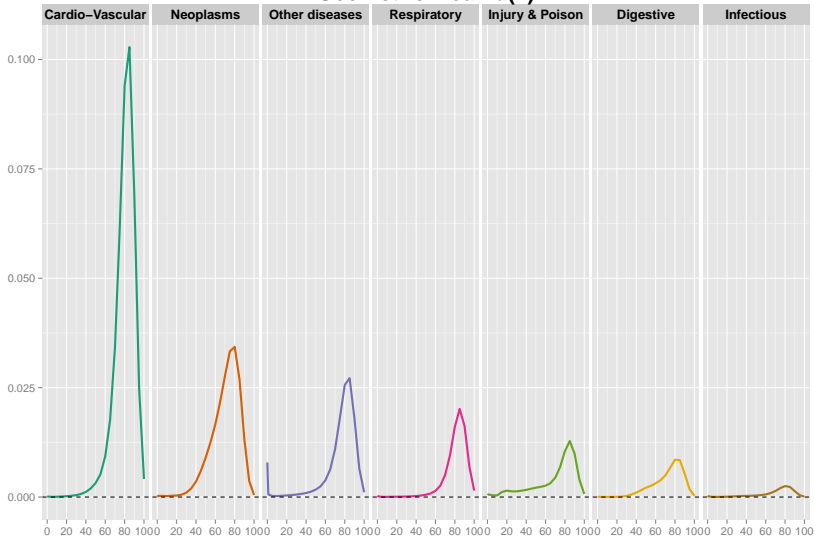


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Deaths by Group of Causes: France, female



Geometric Mean $d(x)$ 

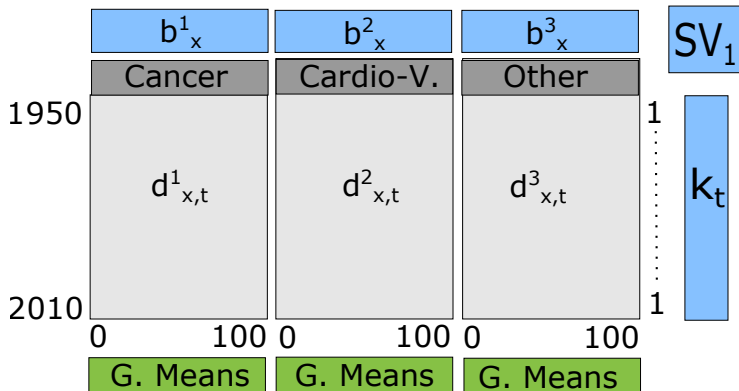
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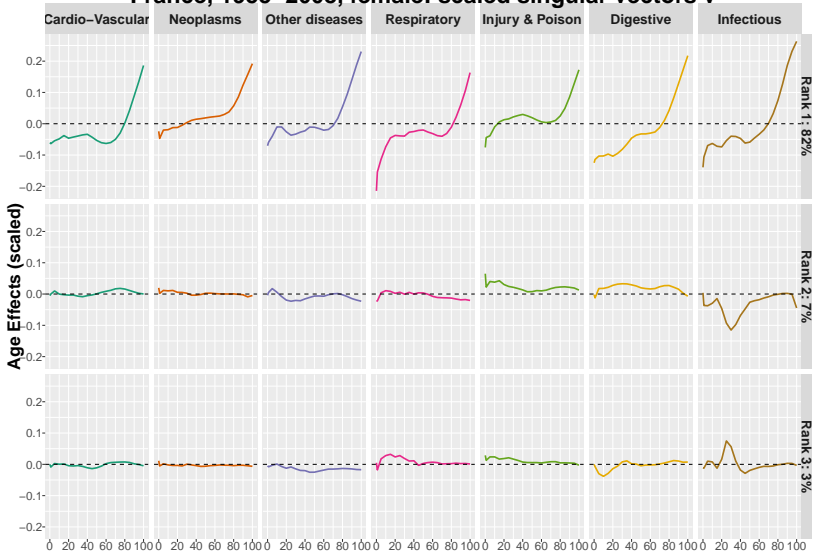
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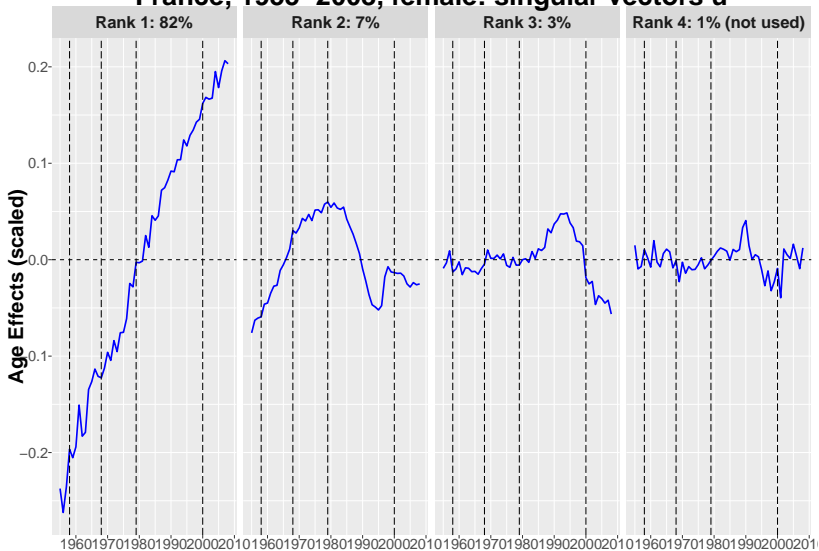
SVD: Multiple Causes of Death



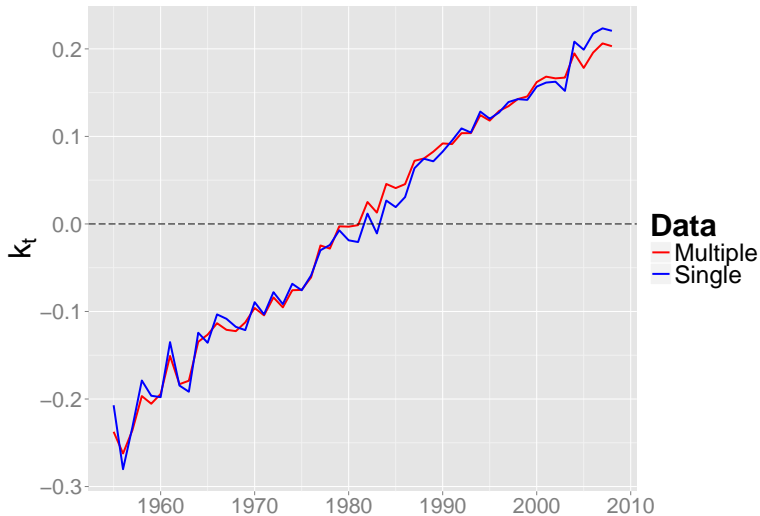
France, 1955–2008, female: scaled singular vectors v



France, 1955–2008, female: singular vectors u



SVD: Time Vectors



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Kullback–Leibler Divergence

$$D_{KL}(P||Q) = \sum_i P_i \log \frac{P_i}{Q_i} \quad (1)$$

$$D_{KL}(P||Q) \geq 0 \quad (2)$$

$$D_{KL}({}_n d_x || \widehat{{}_n d_x}) = \sum_{x=0}^{\omega} {}_n d_x \log \frac{{}_n d_x}{\widehat{{}_n d_x}} \quad (3)$$

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Each Disease versus the Remainder

| Target & Size | | | Across All | | Within Target | |
|---------------|------|-------|------------|------|---------------|------|
| Disease | Rank | Other | D_{KL} | Rank | D_{KL} | Rank |
| All | 0 | | 0.016 | | | |
| C-V | 1 | Rem. | 0.044 | 5 | 0.029 | 2 |
| Neop. | 2 | Rem. | 0.034 | 4 | 0.023 | 1 |
| Other D | 3 | Rem. | 0.031 | 2 | 0.061 | 3 |
| I & P | 4 | Rem. | 0.032 | 3 | 0.072 | 4 |
| Resp. | 5 | Rem. | 0.063 | 6 | 0.130 | 5 |
| Digest. | 6 | Rem. | 0.079 | 7 | 0.159 | 6 |
| Inf. | 7 | Rem. | 0.013 | 1 | 0.322 | 7 |

Neoplasms versus 1 Other and the Remainder

| Target | | Other | | Across All | | Within Target | |
|---------|------|---------|------|------------|------|---------------|------|
| Disease | Rank | Disease | Rank | D_{KL} | Rank | D_{KL} | Rank |
| Neop. | 2 | Rem. | 0 | 0.034 | | 0.023 | |
| Neop. | 2 | C-V | 1 | 0.063 | 5 | 0.027 | 3 |
| Neop. | 2 | Other D | 3 | 0.047 | 1 | 0.024 | 1 |
| Neop. | 2 | I & P | 4 | 0.048 | 2 | 0.025 | 2 |
| Neop. | 2 | Resp. | 5 | 0.061 | 4 | 0.029 | 4 |
| Neop. | 2 | Digest. | 6 | 0.067 | 6 | 0.032 | 6 |
| Neop. | 2 | Inf. | 7 | 0.050 | 3 | 0.029 | 5 |

Best Model by Complexity Level

| Target | Competing Risks | D_{KL} | |
|-----------|--------------------------------------|----------|--------|
| | | Across | Within |
| Neoplasms | Remainder | 0.034 | 0.023 |
| Neoplasms | Other Dis., Remainder | 0.047 | 0.024 |
| Neoplasms | Other Dis., Inj. & Poison, Remainder | 0.050 | 0.024 |

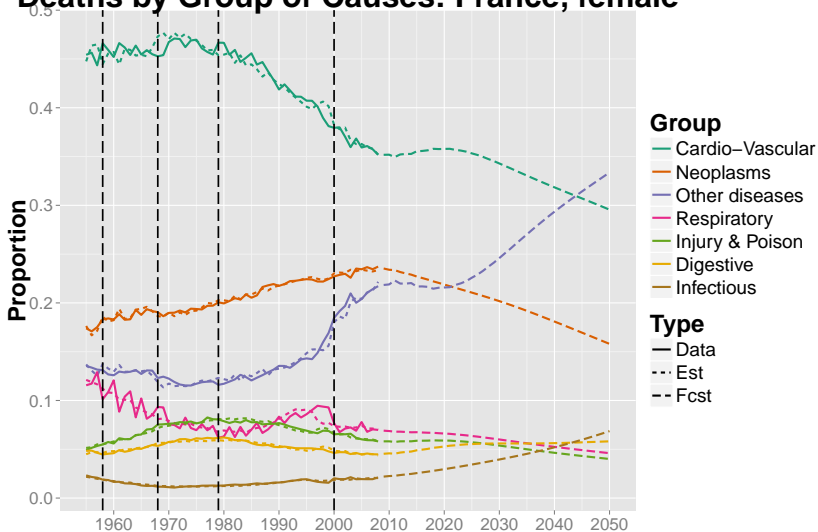
Best Model by Complexity Level

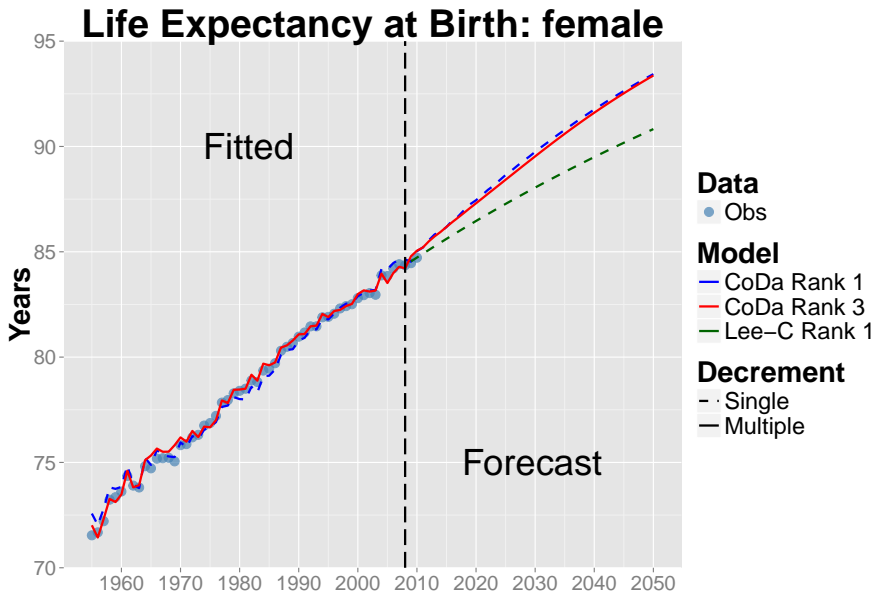
| Target | Competing Risks | D_{KL} | |
|-----------|--------------------------------------|----------|--------|
| | | Across | Within |
| Cardio-V. | Remainder | 0.044 | 0.029 |
| Cardio-V. | Other Dis., Remainder | 0.059 | 0.029 |
| Cardio-V. | Other Dis., Inj. & Poison, Remainder | 0.054 | 0.028 |

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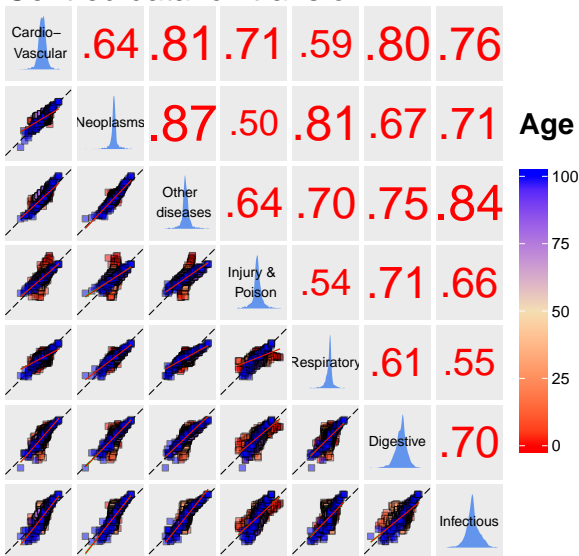




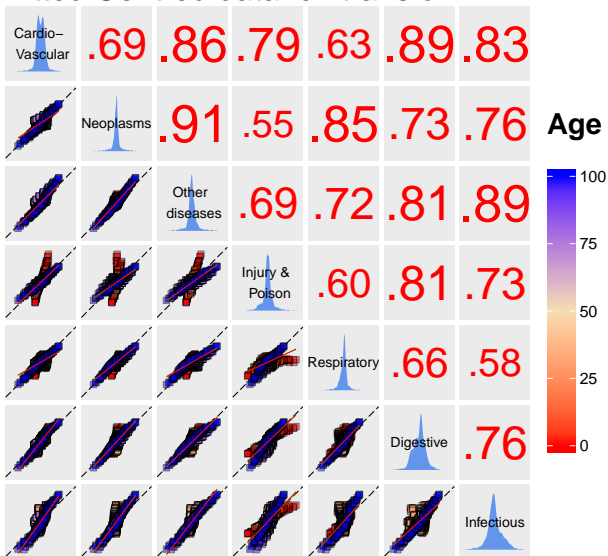
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Centred data: clr transform



Fitted Centred data: clr transform



Compositional residuals: clr transform

